Magnetic Force

Objectives

After going through this lesson, the learners will be able to:

- Derive the expression for force acting on the current carrying conductor in the magnetic field.
- Find the direction of magnetic force
- Derive an expression for force between two long parallel current carrying conductors.
- Define SI unit of current (ampere)

Content Outline

- Unit Syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Force on a current carrying conductor in a uniform magnetic field
- Force between two parallel infinitely long current carrying conductors
- Definition of ampere
- Some Examples
- Summary

Unit Syllabus

Unit –III: Magnetic Effects of Current and Magnetism-10 Modules

Chapter-4: Moving Charges and Magnetism

Concept of magnetic field, Oersted's experiment.

Biot - Savart law and its application to the current carrying circular loop.

Ampere's law and its applications to infinitely long straight wire. Straight and toroidal solenoids, Force on a moving charge in uniform magnetic and electric fields. Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.

Chapter-5: Magnetism and Matter

Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

Para-, dia- and ferro - magnetic substances, with examples. Electromagnets and factors affecting their strengths. Permanent magnets

Module Wise Distribution of Unit Syllabus

Module 1	• Introducing moving charges and magnetism • Direction of magnetic field produced by a moving charge • Concept of Magnetic field • Oersted's Experiment • Strength of the magnetic field at a point due to current carrying conductor • Biot-Savart Law • Comparison of coulomb's law and Biot Savart's law
Module 2	• Applications of Biot- Savart Law to current carrying circular loop, straight wire • Magnetic field due to a straight conductor of finite size • Examples
Module 3	• Ampere's Law and its proof • Application of ampere's circuital law: straight wire, straight and toroidal solenoids. • Force on a moving charge in a magnetic field • Unit of magnetic field • Examples
Module 4	• Force on moving charges in uniform magnetic field and uniform electric field. • Lorentz force • Cyclotron

Module 5	● Force on a current carrying conductor in uniform magnetic field ● Force between two parallel current carrying conductors ● Definition of ampere
Module 6	● Torque experienced by a current rectangular loop in uniform magnetic field ● Direction of torque acting on current carrying rectangular loop in uniform magnetic field ● Orientation of a rectangular current carrying loop in a uniform magnetic field for maximum and minimum potential energy
Module 7	● Moving coil Galvanometer- ● Need for radial pole pieces to create a uniform magnetic field ● Establish a relation between deflection in the galvanometer and the current ● Its current sensitivity ● Voltage sensitivity ● conversion to ammeter and voltmeter ● Examples
Module 8	● Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. ● Torque on a magnetic dipole in a uniform magnetic field. ● Explanation of magnetic property of materials
Module 9	● Dia, Para and ferromagnetic substances with examples. Electromagnets and factors affecting their strengths, permanent magnets.
Module 10	● Earth's magnetic field and magnetic elements.

Module 5

Words You Must Know

- **Coulomb's law:** The force of attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.

- **Electric current:** The time rate of flow of charge in any conductor.
- **Drift Velocity of Electrons:** The average velocity with which electrons move in conductor from the negative end of conductor to the positive end of conductor under the effect of electric field is called drift velocity. The direction of drift velocity is opposite to the direction of flow of current. The relation between current and drift velocity is given by $I = neAV_d$, where *n* is electron density e, charge on electrons, and A, the area of cross section of the conductor.
- **Magnetic field:** The space around a magnet within which its influence can be experienced is called magnetic field.

  Moving charges or a current sets up or creates a magnetic field in the space surrounding it.
- **Magnetic force:** The magnetic field exerts a force on a moving charge or a current in the field.
- **Magnetic force at a point** may be defined as the force acting on a unit charge moving with a unit velocity at right angle to the direction of the field.
- **SI unit of Magnetic field:** SI unit of magnetic field is **tesla (T).** The magnetic field is said to be one tesla if a charge of one coulomb moving with a speed of 1 m/s at right angles to the field experiences a force of one Newton.
- **C G S unit of magnetic field:** cgs unit of magnetic field **is gauss (G).** 1T = 10^{-4} G
- **Magnetic field lines:** It is a curve,, the tangent to which at a point gives the direction of the magnetic field at that point.
- **Maxwell's cork screw rule or right hand screw rule:** It states that if the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.
- **Biot-Savart law:** According to Biot-Savart law, the magnetic field *dB* at *P* due to the current element *Idl* is given by

  $$dB = \frac{\mu_0 Idl sin\theta}{4\pi r^2}$$

  if $\theta = 0^0, sin\,\theta = 0,\ dB = 0$. Magnetic field is 0 at points on the axis of the current element.

  If $\theta = 90^0, sin 90^0 = 1,\ dB\ is\ maximum$. Magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

- **Right hand thumb rule or Curl rule:** If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.
- **Lorentz magnetic force:** The force acting on moving charge in a magnetic field is called Lorentz magnetic force. The force depends upon the angle between the magnetic field and the direction of moving charge.
- **Maximum Force: This force is maximum when the direction of motion of a charged particle is perpendicular to the direction of magnetic field. When the charge particle moves along the direction of magnetic field, it does not experience any force.**: The force acting on the current carrying conductor in magnetic field is maximum when the conductor is placed perpendicular to the direction of magnetic field.
- **Direction of force on current carrying conductor:** Direction of force acting on a current carrying conductor in magnetic field is given by the **right hand palm rule**. The force is perpendicular to both the direction of magnetic field and the direction of current.
- **Electric permittivity** ϵ is a physical quantity that describes how an electric field affects and is affected by a medium. It is determined by the ability of a material to polarise in response to an applied field, and thereby to cancel, partially, the field inside the material.
- **Magnetic permeability** μ is the ability of a substance to acquire magnetisation in magnetic fields. It is a measure of the extent to which magnetic fields can penetrate matter.

  $\epsilon\mu = \frac{1}{v^2}$ **where v is the speed of electromagnetic radiation in the medium.**

Introduction

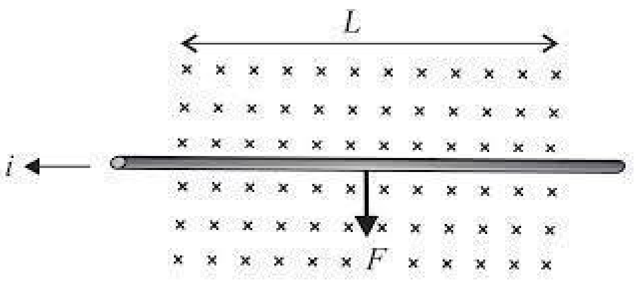
In the previous module, we have discussed that when a charged particle moves in a magnetic field, it experiences a force called **Lorentz magnetic force**. We can now extend the analysis for force due to the magnetic field on a single moving charge to a straight current carrying conductor.

In a conductor, current flows due to drifting of electrons in the conductor. The current carrying conductor when placed in a magnetic field will thus experience force. Of course we can see the mechanical effect of this force on the conductor carrying current, if the conductor is not fixed.

In this module we will discuss in detail about the force on a current carrying conductor and the force between two currents carrying conductors when placed parallel to each other.

Force on a Current Carrying Conductor Placed in Uniform Magnetic field

Consider a conductor of a uniform cross-sectional area A and length ℓ carrying current I be placed in a uniform magnetic field B. The current in a conductor is due to motion of electrons (mobile charge carriers).



Let V_d = drift velocity of electrons.

e = charge on each electron.

Force acting on an electron in the presence of an external magnetic field B.

$\mathbf{F = e\,(V_d \times B)}$

Let n = number of free electrons per unit volume (density of mobile charge carriers).

Therefore, the total number of mobile charge carriers in the conductor.

$\mathbf{N = n\,A\,\ell}$

Hence, the total force on the conductor a

$\mathbf{F = n\,A\,e\,\ell\,(V_d \times B)}$

Now n e V_d is the current density j and [n e A V_d] is the current I flowing in the conductor. Thus

$\mathbf{F = [(j\,A\,\ell)] \times B}$

$\mathbf{F = I\,\ell \times B}$

Since I ℓ is taken as vector element and its direction is identical to the current and I ℓ and V_d are opposite to each other by convention.

If the angle between direction of current and magnetic field is θ, then

$\mathbf{F = I\ell B \sin\theta}$

Special cases:-

(a) When θ = 0 or 180°

Therefore, F = 0

This current carrying conductor experiences no force when placed parallel to the direction of the magnetic field.

(b) If $\theta = 90°$, then $F = I\ell B$ (max)

It means a current carrying conductor experiences a maximum force when placed at right angle to the uniform magnetic field.

Note:-

The equation $F = I\ell \times B$ hold only if the magnetic field is uniform over the entire length of the conductor.

If magnetic field is uniform over a small element of length $d\ell$, then force

$dF = Id\ell \times B$.

Think About

A current carrying conductor in a non-uniform field or a variable current carrying conductor in a uniform magnetic field.

Direction of force magnetic force

The direction of force is perpendicular to the plane containing $I\ell$ and B and can be determined using right hand thumb /palm rule.

https://www.youtube.com/watch?v=3yG_enZqWP8

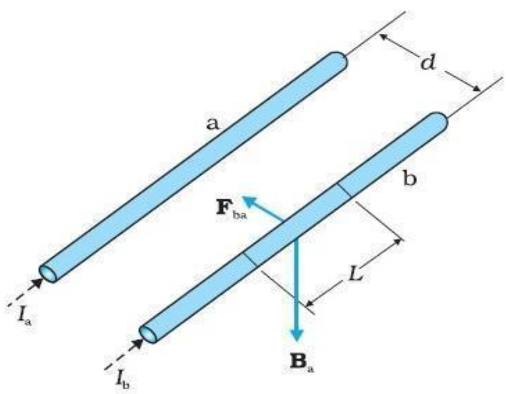
Force Between Two Infinitely Long Parallel Current Carrying Conductors

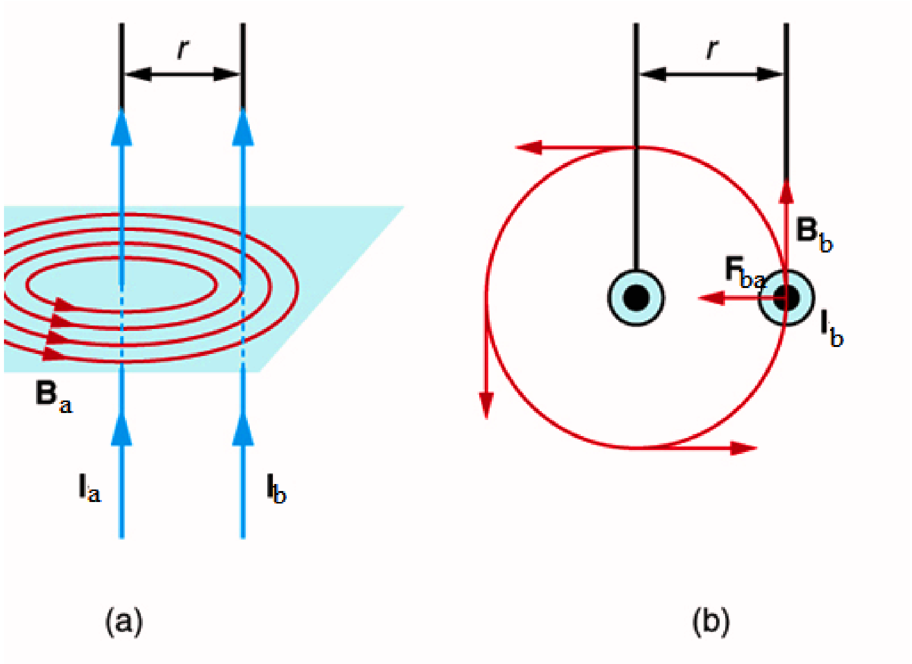
We have learnt that a conductor carrying a current produces a magnetic field. Further, we have learnt that an external magnetic field will exert a force on a current carrying conductor. When the two current carrying conductors placed near each other, will exert magnetic force on each other. Ampere studied the nature of this magnetic force and its dependence on the various parameters. In this section, we shall take the simple example of two parallel current carrying conductors.

Consider two long parallel conductors a and b separated a distance *d* carrying currents I_a and I_b respectively in the same direction.

See the magnetic field lines in figure (a) drawn for conductor carrying current I_a

Notice the force on the second conductor:





The current in the conductor 'a' produces the same magnetic field at all points along the conductor 'b' and its magnitude is given by

$$B_{a}= \frac{\mu_o I_a}{2\pi d}$$

The direction of this field is downward when the conductor is placed horizontally as shown in figure given above.

The conductor 'b' carrying a current I_b will experience a force due to the magnetic field B_a. The magnitude force on conductor 'b' with length L is given by

$$\mathbf{F_{ba} = I_b\, L\, B_a}$$

$$= \frac{\mu_o I_a I_b}{2\pi d} L$$

The direction of this force is towards conductor 'a'

Similarly the current I_b in conductor 'b' produces uniform magnetic field at all points along the conductor 'a' and is given by

$$B_{b}= \frac{\mu_o I_b}{2\pi d}$$

The direction of this field is upward when the conductor is placed horizontally.

Similarly the magnitude of force on conductor 'a' with length L due to the magnetic field B_b is given by

$$\mathbf{F_{ab} = I_a L B_b}$$

$$= \frac{\mu_o I_a I_b}{2\pi d} L$$

This force is equal in magnitude to F_{ba} and directed towards conductor 'b'. Thus

$\mathbf{F_{ba} = - F_{ab}}$

- Therefore we have seen that two conductors carrying currents in the same direction attract each other.
- Similarly we can show that the conductors carrying currents in opposite directions repel each other.

Parallel currents attract, and anti-parallel currents repel.

This rule is the opposite of what we find in electrostatics. Like (same sign) charges repel each other, but like (parallel) currents attract each other.

Definition of Ampere

The magnitude of force acting per unit length between two parallel long straight conductors carrying currents I_a and I_b is given by

$$F = \frac{\mu_o I_a I_b}{2\pi d}$$

If d= 1 m, $I_a = I_b = 1$ A

Therefore

$F = 2 \times 10^{-7}$ N m^{-1}

Thus one Ampere is that current which when flowing through two parallel conductors of infinite length placed in free space at a distance of 1 m apart produces a force of 2×10^{-7} N per unit length between them.

Definition of ampere and the value of μ_0

The ampere is the value of that steady current which , when maintained in each of the two parallel infinitely long straight conductors of negligible area of cross section placed one metre apart in vacuum , would produce a mutual force equal to **2×10^{-7} newton per metre of length.**

This definition of ampere was adopted in **1946,** though not easy to confirm as the condition of removing the effect of earth's magnetic field is difficult.

It is interesting:

- The choice of a force of 2×10^{-7} per metre makes the value of ampere close to the value defined in other ways , like the net charge passing a point per second or through the chemical effect of current

- Considering the expression: $F = \frac{\mu_0 I_1 I_2 L}{2\pi a}$

  If $I_1 = I_2 = 1A$

  $F = 2 \times 10^{-7}$ N/m

  a = 1m

  L = 1m, we have:

  $2 \times 10^{-7} = \frac{\mu_0 \times 1 \times 1 \times 1}{2\pi \times 1}$

  Thus, we have:

  $\mu_0 = 4\pi \times 10^{-7}\ H/m$

  The unit of μ_0 is also NA^{-2}.

  Thus, the definition of ampere assigns a value to the permeability of free space.

This definition of the ampere was adopted in 1946. It is a theoretical definition. In practice one must eliminate the effect of the earth's magnetic field and substitute very long wires by multi turn coils of appropriate geometries. An instrument called the current balance is used to measure the resulting mechanical force.

- The S.I. unit of charge, namely the coulomb can now be defined as in terms of the ampere.

  When a steady current of 1 ampere is set up in a conductor, the quantity of charge that flows through its cross-section in 1s is 1 coulomb (1 C).

Some Examples

Example

Two long straight wires X and Y separated by a distance of 5 cm in air carry currents of 10 A and 5 A respectively in opposite directions.

Calculate magnitude and direction of force on a 20 cm, length of wire Y.

Solution

$$F = \frac{\mu_o 2 I_a I_b}{4\pi d} L$$

$$= \frac{10^{-7}\, 2 \times 10 \times 5}{5 \times 10^{-2}} \times 20 \times 10^{-2}$$

$$= 4 \times 10^{-5}\ N$$

Force is repulsive.

Example

Calculate the force per unit length on a long straight wire carrying current of 4A due to a parallel wire carrying 6A current. Distance between the wires is 3 cm.

Solution

$$F = \frac{\mu_o 2 I_a I_b}{4\pi d} = \frac{10^{-7} 2\times4\times6}{3\times10^{-2}} = \mathbf{1.6 \times 10^{-4}\ N}$$

Example

A straight wire of length L, carrying a current *I* stays suspended horizontally in mid air in a region where there is a uniform magnetic field. The linear mass density of the wire is λ. Obtain the magnitude of the magnetic field.

Solution

Weight of wire = Force exerted by magnetic field

Mg = ILB

λLg = ILB

B = λg/I

Example

What is the magnitude of magnetic force per unit length on a wire carrying a current of 8A and making an angle of 30^0 with the direction of a uniform magnetic field of 0.15T?

Solution

$F = ILB \sin 30^0 = 8 \times 1 \times 0.15 \times 0.5$ **= 0.6 N**

Example

A 3.0 cm wire carrying a current of 10A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T.

What is the magnetic force on the wire?

Hints: F = ILB = $\mathbf{8.1\times10^{-2}\ N}$

Example

Equal currents I is flowing through two infinitely long parallel wires. What will be the magnetic field at a point midway, when the currents are flowing in the same direction?

Hints: Magnetic fields produced by two conductors are equal and opposite in direction. Hence net field is zero.

Example

Equal currents I and I are flowing through two infinitely long parallel wires. What will be the magnetic field at a point midway, when the currents flowing in the opposite direction?

Hints: Magnetic fields produced by two conductors are equal and in the same direction.

Example

Why does a current carrying conductor experience a force in the magnetic field?

Hints: When current flows, free electrons drift in the conductor and experience a force.

Example

Two parallel wires carrying currents in the same direction attract each other. Why?

Hints: Each current carrying conductor produces a magnetic field around it. So each conductor experiences a force due to the magnetic field of the other. Forces are directed towards each other. Draw field lines.

Example

Two parallel wires carrying currents in the opposite direction repel each other. Why?

Hints: Each current carrying conductor produces a magnetic field around it. So each conductor experiences a force due to the magnetic field of the other. Forces are directed away from each other.

Example

A solenoid contract when a current is passed through it?

Hints: currents in the consecutive loops are in the same direction.

Example

A straight wire of mass 200 g and length 1.5m carries a current of 2A. It is suspended in mid-air by a uniform horizontal magnetic field. What is the magnitude of the magnetic field?

Hints: using ILB = mg, we get B = 0.66T, problem solved earlier

Example

Two long and parallel straight wires A and B carrying currents of 8.0A and 5.0A in the same direction are separated by a distance of 4.0cm. Estimate the force on a 10cm section of wire A.

Hints: 2×10^{-5}T

Example

A straight current carrying conductor is placed in a uniform magnetic field. In which orientation the force experienced by the conductor is maximum and minimum?

Hints: The force is minimum (zero) when the conductor is placed parallel to the direction of magnetic field whereas the force is maximum when the conductor is placed perpendicular to the direction of magnetic field.

Example

Two long straight parallel conductors separated by a distance of 20cm carry currents 2A and 4A, respectively in mutually opposite directions. At what point near the conductors, the magnetic field is zero?

Hints: For the magnetic field to be zero, the fields produced by two conductors should be equal in magnitude and opposite in direction. Using this, we can find that the magnetic field is zero at a distance of 20cm outside from the conductor carrying current of 2A.

Example

A long straight wire carries a current *I*. A proton travels with a speed *v* parallel to wire at a distance *d* from it in a direction opposite to the direction of current. In which direction, the proton will experience force?

Hints: The force is perpendicular to the direction of motion of the proton and is directed away from the wire.

Example

There is no change in the energy of a charge particle moving in a magnetic field although a magnetic force is acting on it. Why?

Hints: The magnetic force is perpendicular to the direction of motion. So, no work is done. Thus, there is no change in energy.

Summary

In this module we have learnt that

- When a current carrying conductor is placed in the magnetic field, it experiences a force.
- This force is called Lorentz magnetic force.
- The magnitude of the force depends on the orientation of the conductor in the magnetic field. F = BIL
- When the conductor is parallel to the magnetic field, the conductor experiences no force.
- When the conductor is perpendicular to the magnetic field, the conductor experiences maximum force.
- The direction of this force is always perpendicular to both the direction of current and the magnetic field.
- A force comes into play when two long current carrying conductors are placed parallel to each other.
- The force between conductors is attractive when the currents flow in the same direction in the two conductors. Thus the two conductors carrying currents in the same direction attract each other.
- The two conductors carrying currents in opposite directions repel each other.
- The SI unit of current (ampere). The ampere is the value of that steady current which, when maintained in each of the two parallel infinitely long straight conductors of negligible area of cross section, placed one meter apart in vacuum, would produce a mutual force equal to **2×10^{-7} newton per metre of length.**